

Interference effects in hyperfine induced $2s2s\ ^3P_0, \ ^3P_2 \rightarrow 2s^2\ ^1S_0$ transitions of Be-like ions

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Abstract

The hyperfine induced $2s2p\ ^3P_0, \ ^3P_2 \rightarrow 2s^2\ ^1S_0$ E1 transition probabilities of Be-like ions were calculated using grasp2K based on multi-configuration Dirac-Fock method and HFST packages. It was found that the hyperfine quenching rates are strongly affected by the interference for low- Z Be-like ions, especially for $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ transition. In particular, the trends of interference effects with atomic number Z in such two transitions are not monotone. The strongest interference effect occurs near $Z = 7$ for $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ E1 transition, and near $Z = 9$ for $2s2p\ ^3P_2 \rightarrow 2s^2\ ^1S_0$ E1 transition.

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key words: Hyperfine induced transition; Hyperfine spectroscopy; Interference effect; Be-like ions.

1 Introduction

Hyperfine spectroscopy is very important tool in study of atomic and nuclear physics, especially which can be used to check fundamental interaction [1, 2, 3, 4, 5] such as electromagnetic and electroweak interaction with high accuracy, determine nuclear properties

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[6, 7, 8, 9, 10], develop atomic clock [11, 12, 13, 14, 15], and so on. Recently one kind of hyperfine transitions, which is known as hyperfine induced transition or hyperfine quenching, attracts more attention owing to analyzing spectra [16], determining isotopic ratios in stellar and diagnosing low-density plasma [17, 18] besides those reasons mentioned above.

Many theoretical researches have been made to predict hyperfine induced transition probability based on different method. The most important problem in the calculations of hyperfine quenching rate is how to treat the hyperfine interaction and the interaction with electromagnetic field. In the past decades, three methods have been developed to solve such problem, i.e. perturbative method [19, 20], complex matrix method [21, 22] and radiative damping method [23, 24]. They have individual merit and shortage in actual physical problems as shown in Ref. [23, 24]. Here we used perturbative method to evaluate hyperfine quenching rate, that is, the computations of the probability follow the determinations of hyperfine structure. This approach is valid as long as radiative widths are smaller than the fine separation between concerned levels. The crucial point is to properly choice perturbative states involved in because the differences of results calculated by different approximate model are sometimes very large.

For hyperfine induced $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ transition of Be-like ions there have been several studies. Marques *et al.* firstly computed the probabilities using their developing complex matrix method [25]. However, in their computational model they neglected an important contribution from the $2s2p\ ^1P_1$ level, which lead to a relative large discrepancy compared to the later theoretical [17, 24] and experimental value [18, 26]. Once again, Brage *et al.* gave some of transition rates by means of perturbative method in order to determine the isotopic of composition and diagnose densities of low-density plasmas[17]. In their investigation the influence of $2s2p\ ^1P_1$ level on hyperfine induced $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ transition of Be-like ions were indicated. But the calculations were restricted within those abundant elements in stellar. Later, Schippers *et al.* measured the probability for Be-like Ti ions using resonant electron-ion recombination method in the heavy-ion storage ring TSR of Max-Planck Institute for Nuclear Physics, Heidelberg, Germany [26]. The experimental result was almost 60% larger than Marques *et al.* theoretical value. This led to Cheng *et al.* renewedly calculating this decay rate along Be-like isoelectronic sequence by

perturbative and radiative damping method [24]. The latest theoretical results reduced the discrepancy to 20%. Their investigations shown again that the contribution from $2s2p\ ^1P_1$ transition amplitude and the interference effect from $2s2p\ ^3P_1$ and $2s2p\ ^1P_1$ perturbative states on hyperfine induced $2s2s\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ transition of Be-like ions can not be neglected. Even though, it seems that characteristic about interference effects in hyperfine induced $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ transition of Be-like ions were still not very clear to be revealed.

As we know, 3P_2 level is another metastable state of sp configuration. It can decay to the excited state 3P_1 by magnetic dipole (M1) transition and to the ground state by magnetic quadrupole (M2) transition with large branch ratio. Many publications have been concerned with determination of the probabilities [27, 28, 29, 30, 31] to diagnose the low-density plasma and to probe relativistic and QED effects by accurate transition energy and probability. However, information about another transition process, that is, hyperfine induced E1 transition is very scarce. While, Gould *et al.* and Andersson *et al.* have pointed out significant influences of hyperfine induced E1 transition on the lifetime of 3P_2 level in He-like [32] and Zn-like ions [33], respectively. Dubau *et al.* have also shown that quantum interference between the E1 transition of hyperfine induced and M2 transition has obvious effects on increasing the degree of linear polarization of $^3P_2 \rightarrow ^1S_0$ in He-like ions [34], which can affect the modelling and diagnostics of high-temperature astrophysical and laboratory plasmas with an anisotropic non-Maxwellian velocity distribution of energetic electrons.

Based on these reasons mentioned above, we further investigated hyperfine induced $2s2p\ ^3P_2, ^3P_0 \rightarrow 2s^2\ ^1S_0$ transition of Be-like ions in detail using grasp2K [35] based on multi-configuration Dirac-Fock and HFST [36] packages. The regular of interference effects for hyperfine induced $2s2s\ ^3P_0, ^3P_2 \rightarrow 2s^2\ ^1S_0$ transition of Be-like ions was shown. In particular, it was found that the trends of interference effects with atomic number Z in such two transitions are not monotone.

2 Theory

2.1 Wavefunction of hyperfine level

In the present of hyperfine interactions, which couple the nuclear I and electronic J angular momenta to a total angular momenta $F = I + J$, only F and M_F are good quantum number other than the electronic angular momenta J and M_J . Then, the wave function for the system can be written by

$$|FM_F\rangle = \sum_{\gamma J} h_{\gamma J} |\gamma J I F M_F\rangle, \quad (1)$$

where $h_{\gamma J}$ is the mixing coefficients due to hyperfine interaction, and are obtained in first order perturbation theory as the ratio between the off-diagonal hyperfine matrix elements and the unperturbed energy differences

$$h_{\gamma J} = \frac{\langle \gamma J I F M_F | H_{hfs} | \gamma_0 J_0 I F M_F \rangle}{E(\gamma_0 J_0) - E(\gamma J)}, \quad (2)$$

the subscript 0 labels the concerned level. The hyperfine interaction Hamiltonian H_{hfs} in this formula can be represented as a multipole expansion [37, 38],

$$H_{hfs} = \sum_{k \geq 1} \mathbf{T}^{(k)} \cdot \mathbf{M}^{(k)}, \quad (3)$$

where $\mathbf{T}^{(k)}$ and $\mathbf{M}^{(k)}$ are spherical tensor operators of rank k in the electronic and nuclear spaces, respectively [37]. In the following discussion we only include main the nuclear magnetic dipole ($k=1$) and electric quadrupole ($k=2$) interaction. Applying Racah's algebra, hyperfine interaction matrix elements can be further written by [39]

$$\langle \gamma J I F M_F | H_{hfs} | \gamma_0 J_0 I F M_F \rangle = (-1)^{I+J_0-F} [(2J+1)(2I+1)]^{1/2} \begin{Bmatrix} I & J & F \\ J_0 & I & 1 \end{Bmatrix} \langle \gamma J || \mathbf{T}^{(1)} || \gamma J_0 \rangle \langle I || \mathbf{M}^{(1)} || I \rangle, \quad (4)$$

$$\langle \gamma J I F M_F | H_{hfs} | \gamma_0 J_0 I F M_F \rangle = (-1)^{I+J_0-F} [(2J+1)(2I+1)]^{1/2} \begin{Bmatrix} I & J & F \\ J_0 & I & 2 \end{Bmatrix} \langle \gamma J || \mathbf{T}^{(2)} || \gamma J_0 \rangle \langle I || \mathbf{M}^{(2)} || I \rangle. \quad (5)$$

The reduce matrix elements of the tensor $\mathbf{M}^{(k)}$ are related to the conventionally defined nuclear moment,

$$\langle I || \mathbf{M}^{(1)} || I \rangle = \mu_I \sqrt{\frac{I+1}{I}}, \quad (6)$$

$$\langle I || \mathbf{M}^{(2)} || I \rangle = \frac{Q}{2} \sqrt{\frac{(2I+3)(I+1)}{I(2I-1)}}. \quad (7)$$

where μ_I is nuclear magnetic dipole moment in μ_N of the nuclear magneton, and Q is electric quadrupole moment in barns.

2.2 Hyperfine transition probability

The electric dipole (E1) transition probability between two different hyperfine levels $|FM_F\rangle$ and $|F'M'_F\rangle$ is given by [45]

$$A = \frac{4\omega^3}{3c^3} \sum_{M_F} |\langle FM_F | \mathbf{Q}^{(1)} | F'M'_F \rangle|^2, \quad (8)$$

where $\mathbf{Q}^{(1)}$ is the electric dipole tensor operator. Substitute (1) into above formula, then

$$A = \frac{4\omega^3}{3c^3} \frac{1}{2F'+1} \left| \sum_{\gamma J} \sum_{\gamma' J'} h_{\gamma J} h_{\gamma' J'} \langle \gamma J I F || Q^{(1)} || \gamma' J' I F' \rangle \right|^2. \quad (9)$$

Because operator $Q^{(1)}$ only act on electronic parts, reduced matrix element $\langle \gamma J I F || Q^{(1)} || \gamma' J' I F' \rangle$ can be simplified based on Racah's algebra [39],

$$\langle \gamma J I F || Q^{(1)} || \gamma' J' I F' \rangle = (-1)^{(J+I+F')} \sqrt{(2F+1)(2F'+1)} \left\{ \begin{matrix} J & F & I \\ F' & J' & 1 \end{matrix} \right\} \langle \gamma J || Q^{(1)} || \gamma' J' \rangle, \quad (10)$$

therefore,

$$A = \frac{4\omega^3}{3c^3} (2F+1) \left| \sum_{\gamma J} \sum_{\gamma' J'} h_{\gamma J} h_{\gamma' J'} \left\{ \begin{matrix} J & F & I \\ F' & J' & 1 \end{matrix} \right\} \langle \gamma J || Q^{(1)} || \gamma' J' \rangle \right|^2, \quad (11)$$

where ω is the transition energy in Hartree. The reduced transition matrix elements of the electric dipole operator can be obtained as square roots of the corresponding line strengths.

Using similar method one can obtain other type hyperfine induced transition probability such as M1, E2, etc. [5, 23, 40].

As can be seen from the derivation, hyperfine transition probability depends on nuclear parameters. It is not convenience for us to further discuss the trend of the rate along atomic number Z . Therefore, we generalized Brage *et al.* method [17] so that hyperfine

transition rate is independent of nuclear properties, which is called reduced hyperfine transition probability A^{el} . By defining reduced hyperfine mixing coefficient h^{el} ,

$$h^{el} = \frac{(-1)^{-(I+J_0+F)}h}{\mu_I[(1+I^{-1})(2I+1)]^{1/2}W(IJ_0 \ II; F1)}, \quad (12)$$

then,

$$A^{el} = \frac{4\omega^3}{3c^3}(2F+1) \left| \sum_{\gamma J} \sum_{\gamma' J'} h_{\gamma J}^{el} h_{\gamma' J'}^{el} \begin{Bmatrix} J & F & I \\ F' & J' & 1 \end{Bmatrix} \right| \langle \gamma J || Q^{(1)} || \gamma' J' \rangle|^2 \quad (13)$$

where $W(IJ_0 \ II; F1)$ are 6j-symbol in eq(4). To simplify we neglected electric quadrupole hyperfine interaction in above equation due to quite weak compared to the magnetic dipole interaction.

2.3 Electronic wave function

The electronic wave functions $|\gamma J\rangle$ were computed using the grasp2K program package [35]. Here the wave function for a state labeled γJ is approximated by an expansion over jj -coupled configuration state functions (CSFs)

$$|\gamma J\rangle = \sum_j c_j \Phi_j. \quad (14)$$

The configuration state functions Φ_j are anti-symmetrized linear combinations of products of Dirac orbitals. In the multi-configuration self-consistent field (SCF) procedure both the radial parts of the orbitals and the expansion coefficients are optimized to self-consistency. In the present work a Dirac-Coulomb Hamiltonian was used with the nucleus described by an extended Fermi charge distribution.

Once the radial orbitals have been determined relativistic configuration interaction (CI) calculations can be performed. Here higher-order interactions may be included in the Hamiltonian. The most important of these is the Breit interactions

$$H_{Breit} = - \sum_{i < j}^N \left[\frac{\alpha_i \cdot \alpha_j \cos(\omega_{ij} r_{ij})}{r_{ij}} + (\alpha_i \cdot \nabla_i)(\alpha_j \cdot \nabla_j) \frac{\cos(\omega_{ij} r_{ij}) - 1}{\omega_{ij}^2 r_{ij}} \right], \quad (15)$$

where photon frequency ω_{ij} is obtained as the difference between the diagonal Lagrange multipliers ϵ_i and ϵ_j associated with the orbitals. However, this is invalid when shells are

multiply occupied, and the diagonal energy parameters of correlation orbitals with small occupation numbers may be large positive quantities totally unrelated to binding energies [41, 42]. For this reason, the zero-frequency limit have been adopted in present calculations. In the configuration interaction calculations the main quantum electrodynamics (QED) effects can also be included.

Tensor algebra used for evaluating hyperfine and electric dipole matrix elements between CI wave functions assumes that the wave functions are built from a common orbital set. This is a severe restriction since high-quality wave functions demands orbitals optimized for the specific state. To relax this and to be able to compute matrix elements between wave functions built from independently optimized orbital sets, biorthogonal transformation techniques introduced by Malmqvist can be used [43, 44].

3 Results and discussions

3.1 Calculational model and method

The accuracy of the calculated hyperfine induced transition rate depends on the number of perturbative states in Eq. (11) and on the accuracy of the electronic matrix elements. In practical calculation of hyperfine induced E1 transition probability for $2s2p\ ^3P_0, \ ^3P_2 \rightarrow 2s^2\ ^1S_0$ Eq. (11) is can be simplified to

$$A = \frac{4\omega^3}{9c^3} \left| \sum_{S=0,1} h_S \langle 2s^2\ ^1S_0 \| Q^{(1)} \| 2s2p\ ^{(2S+1)}P_1 \rangle \right|^2, \quad (16)$$

The differences for these two transitions concentrate on different hyperfine mixing coefficient h_S and transition energy ω . As can be seen from Eq. (16), there exist interference effects caused by the 3P_1 and 1P_1 two transition amplitudes.

The accuracy of the electronic matrix elements involved in above formula was determined by wavefunction of electronic part. The main uncertainty in calculation of the wavefunctions comes from electron correlation effects, especially for low charged ions. For instance C^{2+} ions, those matrix elements are fair sensitive to electronic correlation effects [46, 47, 49]. Therefore, it must be considered very carefully. As a starting point SCF calculations were done for the configurations in the even and odd state complex. In the

calculations the wavefunctions of $1s^2 2s^2 \ ^1S_0$, $1s^2 2s 2p \ ^3P_{0,1,2}$ and $1s^2 2s 2p \ ^1P_1$ were determined in extended optimal level (EOL) calculations [50], respectively. These calculation were followed by calculations with expansions including configuration state functions obtained by single(S) and double(D)-excitations from, respectively, the studied even and odd state reference configurations to active sets of orbitals $n \leq 5$. While for C^{2+} the active set was expanded to $n \leq 7$, and for N^{3+} and F^{5+} to $n \leq 6$ in order to obtain satisfied results. The active sets were systematically increased allowing computed properties to be monitored. Due to stability problems in the relativistic SCF procedure only the outermost layers of orbitals could be optimized each time. The SCF calculations were followed by CI calculations in which part of core-valence and core-core correlations, the frequency independent Breit interaction and QED effects was included. Finally, the hyperfine induced transition probability can be obtained by above mentioned theoretical method using HFST package [36].

3.2 Hyperfine quenching rate

In Table 1 we listed our calculated results for hyperfine induced transition probability as well as other theoretical and experimental value[17, 24, 25, 26]. Nuclear parameters of concerned isotopes for Be-like ions were taken from [51]. As can be seen from Table 1, results of Marques *et al.* obviously deviate from others because two problems. One is that they neglected the contribution of 1P_1 to hyperfine quenching rate of 3P_0 , and the other is that a ratio factor in transition rate was omitted in their method [24]. The present calculational results displayed in the forth column in Table 1 are in agreement with others for $Z \leq 30$, but not for ions with $Z > 30$. The reason for this is we neglected high order Breit interactions and QED effects in electronic wavefunction calculations, which lead to quite large errors in transition energy for high-Z ions. Due to limits of present method, we have not intrinsically solved this problem. Hence experimental [52] and some of other accurate theoretical transition energies [24] were used to correct the hyperfine induced transition probability of 3P_0 levle. Those corrected ones were presented in the fifth column of Table 1. It can be found that the consistence becomes obviously better with Brage *et al.* and Cheng *et al.* calculational value. While for Be-like ^{103}Rh ion

the two order of magnitude difference is because different magnetic dipole moment was used. Based on this, we used the same method to correct other transition rates in next computations as well.

In Table 2 we presented hyperfine quenching rate of 3P_2 in connection with corresponding transition energies used to correct those rates. Since angular momentum J of 3P_2 state unequal zero, it splits into several hyperfine levels, labeled by total angular momentum F . For individual hyperfine level satisfying select rule of electric dipole can occur hyperfine induced E1 transition, and this transition rate is dependent on angular number F and nuclear parameters.

Due to lack of other results about hyperfine induced $2s2p\ ^3P_2 \rightarrow 2s^2\ ^1S_0$ transition probability we can not make comparison. In order to confirm the rates is reliable, therefore, we further evaluated $2s2p\ ^3P_2 \rightarrow 2s^2\ ^1S_0$ M2 and $2s2p\ ^3P_2 \rightarrow 2s2p\ ^3P_1$ M1 transition probabilities using our calculated line strength and experimental transition energies. These results as well as other theoretical values were displayed in Table 3. As can be seen from this Table, the consistence among these results is quite good. It is indicated that our calculational hyperfine quenching rates of 3P_2 level are credible.

3.3 Interference effects in hyperfine quenching

Brage *et al.* and Cheng *et al.* have dictated that interference effects strongly affect hyperfine quenching rate of 3P_0 [17, 24]. From Eq.(16), we know that this effect occur in hyperfine quenching of 3P_2 as well. To show clearly this phenomena, independent nuclear parameter of the transition amplitudes contributed from 3P_1 and 1P_1 were plotted in Fig. 1. As can be seen from this picture, the interference effects work within a wide range of atomic number due to non-monotone change of transition amplitude contributed by 1P_1 . It is interesting that the trend of transition amplitude with Z is similar between those two hyperfine quenching. An obvious difference is that the transition amplitude of 1P_1 is dominant in $2s2p\ ^3P_2 \rightarrow 2s^2\ ^1S_0$ E1 transition while 3P_1 for $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ E1 transition.

In order to reveal characteristic of the interference effects in such hyperfine induced $2s2p\ ^3P_0, ^3P_2 \rightarrow 2s^2\ ^1S_0$ transition of Be-like ions, it is convenience to define a function,

R^{el} , that proportion to the ratio between the two transition amplitudes. For example, for $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ transition,

$$R^{el}(^3P_0) = \left| \frac{h_1 \langle 2s^2\ ^1S_0 || Q^{(1)} || 2s2p\ ^3P_1 \rangle}{h_0 \langle 2s^2\ ^1S_0 || Q^{(1)} || 2s2p\ ^1P_1 \rangle} \right| - 1. \quad (17)$$

According to the formula, the closer R^{el} is to 0, the stronger interference effect is. Therefore, it clearly show the extent of interference effects. The trend of R^{el} with Z for 3P_0 and 3P_2 is plotted in Fig. 2, respectively. It worth noting from this picture that the interference effects in these two hyperfine quenching do not change monotonically and there exist minimum value for $R^{el}(^3P_0)$ near $Z = 7$ and near $Z = 9$ for $R^{el}(^3P_2)$. Hence, the strongest interference effect occurs near $Z = 7$ for $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ E1 transition, and near $Z = 9$ for $2s2p\ ^3P_2 \rightarrow 2s^2\ ^1S_0$ E1 transition.

4 Conclusion

In conclusion, we have calculated the hyperfine induced $2s2p\ ^3P_0, ^3P_2 \rightarrow 2s^2\ ^1S_0$ E1 transition probability of Be-like ions using grasp2K based on multiconfiguration Dirac-Fock method and HFST packages. The interference effects resulted from 3P_1 and 1P_1 perturbative states in those two hyperfine quenching of $2s2p\ ^3P_0, ^3P_2 \rightarrow 2s^2\ ^1S_0$ were studied in detail. It worth noting that the trends of interference effects with atomic number Z in such two transitions are not monotone. The strongest interference effect occurs near $Z = 7$ for $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ E1 transition, and near $Z = 9$ for $2s2p\ ^3P_2 \rightarrow 2s^2\ ^1S_0$ E1 transition.

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6 Figure caption

Fig. 1. Transition amplitude of reduced hyperfine induced transition in a.u.. Left: $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ transition; Right: $2s2p\ ^3P_2 \rightarrow 2s^2\ ^1S_0$ transition.

Fig. 2. The trend of R^{el} for 3P_0 and 3P_2 level with atomic number Z. Two blue arrows label the positions where inference effects are the strongest for the hyperfine induced $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ and $2s2p\ ^3P_2 \rightarrow 2s^2\ ^1S_0$ transition, respectively.

Table 1: Hyperfine induced $2s2p\ ^3P_0 \rightarrow 2s^2\ ^1S_0$ transition probability in s^{-1} and corresponding transition energy in cm^{-1} . The calculational results were compared with other theoretical and experimental value. uncorr. means the hyperfine induced transition probabilities was computed using present calculational transition energy, while the corr. used the ones from NIST database [52].

ions	Transition energy		This work		Ref.[17]	Ref.[24]	Ref.[25]	Expt.
	This work	NIST [52]	uncorr.	corr.				
^{13}C	52248	52367	8.28[-4]	8.33[-4]	9.04[-4]	8.223[-4]	2.00[-4]	
^{14}N	67251	67209	4.40[-4]	4.39[-4]	4.92[-4]	4.40[-4]	1.28[-4]	$4[-4]\pm 1.32^a$
^{19}F	96666	96590	1.17[-1]	1.17[-1]		1.208[-1]	3.60[-2]	
^{28}Si	169054	169802	5.89[-2]	5.97[-2]	6.08[-2]	6.011[-2]	2.16[-2]	
^{39}Ar	228716	228674	8.28[-1]	8.27[-1]				
^{47}Ti	289562	288190	6.80[-1]	6.71[-1]		6.727[-1]	3.56[-1]	$5.6[-1]^b$
^{57}Fe	352029	348180	4.98[-2]	4.82[-2]	5.45[-2]	4.783[-2]	3.27[-2]	
^{67}Zn	416600	409827 [†]	5.00	4.76		4.732	4.13	
^{85}Rb	537174	523000	43.3	39.94		39.35	48.17	
^{103}Rh	693209	661772 [†]	147.1	128.0		1.262	1.91	
^{131}Xe	903919	843105 [†]	199.0	161.5		158.1	262.67	

[†] Cheng *et al.* [24]

^a Brage *et al.* [18]

^b Schippers *et al.* [26]

Table 2: Hyperfine induced $2s2p\ ^3P_2 \rightarrow 2s^2\ ^1S_0$ E1 transition probability A in s^{-1} associated with corresponding reduced transition rate A^{el} in s^{-1} and transition energy ΔE from NIST database [52] in cm^{-1} .

ions	ΔE	A^{el}	F	A	ions	ΔE	A^{el}	F	A
^{13}C	52447	9.87[-4]	3/2	7.30[-4]	^{57}Fe	471780	1.24	3/2	1.52[-1]
			5/2	0				5/2	0
^{14}N	67412	2.52[-4]	1	2.08[-4]	^{67}Zn	640470	4.40	1/2	0
			2	3.69[-4]				3/2	8.81
			3	0				5/2	1.75[1]
^{19}F	97437	1.21[-3]	3/2	1.26[-1]	^{85}Rb	1094800	3.83[1]	7/2	1.79[1]
			5/2	0				9/2	0
^{28}Si	177318	1.70[-2]	3/2	7.68[-2]	^{103}Rh	2310547 [†]	4.08[2]	1/2	0
			5/2	0				3/2	1.85[2]
^{39}Ar	252683	8.28[-2]	3/2	0	^{131}Xe	3785850	4.96[3]	5/2	3.65[2]
			5/2	5.83[-1]				7/2	3.71[2]
			7/2	1.03				9/2	0
			9/2	9.76[-1]				3/2	4.79[2]
			11/	0				5/2	0
^{47}Ti	347420	3.34[-1]	1/2	0				1/2	3.95[3]
			3/2	4.82[-1]				3/2	1.30[4]
			5/2	1.03				3/2	1.30[4]
			7/2	1.17					
			9/2	0					

[†] Cheng *et al.* [24]

Table 3: $2s2p\ ^3P_2 \rightarrow 2s^2\ ^1S_0$ M2 and $2s2p\ ^3P_2 \rightarrow 2s2p\ ^3P_1$ M1 transition probabilities of Be-like ions in s^{-1} connecting with corresponding transition energy from NIST database [52] in cm^{-1} . The calculational results were compared with other theoretical and experimental value

ions	M2 ($^3P_2 - ^1S_0$)				M1($^3P_2 - ^3P_1$)				
	Transition energy	This work	Ref. [30]	Ref. [31]	Transition energy	This work	Ref. [28]	Ref. [29]	Ref. [53]
^{13}C	52447	5.13[-3]	5.190[-3]	5.176[-3]	56	2.37[-6]	2.34[-6]		2.446[-6]
^{14}N	67416	1.14[-2]	1.154[-2]	1.147[-2]	144	4.03[-5]	3.93[-5]		4.070[-5]
^{19}F	97437	3.64[-2]	3.678[-2]	3.633[-2]	587	2.73[-3]			
^{28}Si	177318	2.42[-1]	2.431[-1]	2.410[-1]	5174	1.87	1.83		
^{39}Ar	252683	7.89[-1]	7.904[-1]	7.858[-1]	16820	6.39[1]	6.41[1]	6.417[1]	
^{47}Ti	347240	2.44		2.4234	42620	1.03[3]		1.0369[3]	
^{57}Fe	471780	7.68	7.652	7.6459	92655	1.04[4]	1.11[4]		
^{67}Zn	640470	2.54[1]		2.5289[1]	180855	7.52[4]			
^{85}Rb	1094800	2.28[2]			480900	1.31[6]			
^{103}Rh	1996313 [†]	2.90[3]		2.8789[3]	1198404 [†]	1.86[7]			
^{131}Xe	3785850	4.62[4]			2758850	2.10[8]			

[†] Cheng *et al.* [24]



